Name Klasse Datum Seite Blatt

AP 2001 – AI

1.1. $f(-x) = 2 \cdot \frac{1+\ln((-x)^2)}{-x} = -2 \cdot \frac{1+\ln(x^2)}{x} = -f(x)$

$\Rightarrow$ PSym z. Urspr.

$f(x) = 2 \cdot \frac{1+\ln(x^2)}{x} = 0 \iff 1+\ln(x^2) = 0$

$\ln(x^2) = -1 \iff \ln(x^2) = -\ln(e) = \ln\left(\frac{1}{e}\right) \iff x_{1/2} = \pm\frac{1}{\sqrt{e}}$ Sym!?

1.2 $x \to 0^+ : 2 \frac{1+\ln(x^2) \to -\infty}{x \to 0^+} \to -\infty$; $x \to 0^- : f(x) \to +\infty$ Sym.

$x \to +\infty : 2 \cdot \frac{1+\ln(x^2) \to +\infty}{x \to +\infty} \xrightarrow{L.H} 2 \cdot \frac{\frac{1}{x^2} \cdot 2x}{1} = \frac{4}{x} \to 0^+$

$x \to -\infty : f(x) \to 0^-$ (Sym.)

1.3. $f'(x) = 2 \cdot \frac{\overbrace{x \cdot \frac{1}{x^2} \cdot 2x}^{=2} - (1+\ln(x^2)) \cdot 1}{x^2} = 2 \frac{1-\ln(x^2)}{x^2}$

$f''(x) = 2 \cdot \frac{x^2\left(-\frac{1}{x^2} \cdot 2x\right) - (1-\ln(x^2)) \cdot 2x}{x^2} = 2 \cdot \frac{-2x - (1-\ln(x^2)) \cdot 2x}{x^{2} \; 1}$

$= 2 \frac{-2 - 2(1-\ln(x^2))}{x} = 4 \cdot \frac{-2+\ln(x^2)}{x} = 4 \cdot \frac{\ln(x^2)-2}{x}$

1.4 Extrema: $f'(x) = 2 \cdot \frac{1-\ln(x^2)}{x} = 0 \iff 1-\ln(x^2) = 0$

$\iff \ln(x^2) = 1 \iff x^2 = e \iff x_{1/2} = \pm\sqrt{e}$ $(\approx \pm 1.65)$

$f(\sqrt{e}) = 2 \cdot \frac{1+\ln(e)}{\sqrt{e}} = 2 \cdot \frac{1+1}{\sqrt{e}} = \frac{4}{\sqrt{e}} \approx 2,43$

$f''(\sqrt{e}) = 4 \cdot \frac{\ln(e)-2}{e\sqrt{e}} = 4 \cdot \frac{1-2}{e\sqrt{e}} < 0 \Rightarrow$ HOP$\left(\sqrt{e} \,\middle|\, \frac{4}{\sqrt{e}}\right)$

TIP$\left(-\sqrt{e} \,\middle|\, -\frac{4}{\sqrt{e}}\right)$ wegen Sym.

WEP: $f''(x) = 0 \Rightarrow \ln(x^2)-2 = 0 \iff \ln(x^2) = 2 \iff x^2 = e^2$

$x_{1/2} = \pm e$ einfach, weil die NST von ln m. VZW sind $\Rightarrow$ WEP

$f(e) = 2 \cdot \frac{1+\ln(e^2)}{e} = 2 \cdot \frac{1+2}{e} = \frac{6}{e}$; WEP$_1\left(e; \frac{6}{e}\right)$

WEP$_2\left(-e \,\middle|\, -\frac{6}{e}\right) \approx$ WEP$_2(-2,7 \,|\, -3,64)$ wegen Sym.

Name

AP 2001 – AI

Klasse | Datum | Seite | Blatt

2.0 $g_a(x) = (1-a)x + \frac{2}{x} \quad ; \quad a \in \mathbb{R}$

2.1

$g_a(x) = (1-a)x + 2\cdot x^{-1} \Rightarrow g_a'(x) = 1-a-1\cdot 2x^{-2} = 1-a-\frac{2}{x^2}$

$g_a'(x) = \frac{(1-a)x^2-2}{x^2} = 0 \Rightarrow x^2 = \frac{2}{1-a} \quad ; \quad a \neq 1$

1\. Fall : $1-a < 0 \Leftrightarrow a > 1$: keine waagr. Tang.

2\. Fall : $a = 1 \Rightarrow g_1'(x) = -\frac{2}{x}$: keine waagr. Tang.

3\. Fall : $a < 1$: zwei Stellen $x_{1/2} = \pm\sqrt{\frac{2}{1-a}}$ m. waagr. Tang.

(m. VZW ⇒ HOP/TIP)

2.2. $g_1(x) = \frac{2}{x} = 2\cdot\frac{1+\ln(x^2)}{x} = f(x)$ Graph → Geogebra

$\Leftrightarrow 2 = 2+2\ln(x^2) \Leftrightarrow \ln(x^2) = 0 \Leftrightarrow x^2 = 1 \Leftrightarrow x_{1/2} = \pm 1$ Sym!

2.3 z.z. $F(x)' = (\ln^2(-x)+C)' = \frac{\ln(x^2)}{x}$ für $x<0$

$F'(x) = 2\cdot\ln(-x)\cdot\frac{1}{-x}\cdot(-1) = 2\ln(-x)\cdot\frac{1}{x} = \frac{\ln[(-x)^2]}{x}$ n. diff!

$= \frac{\ln(x^2)}{x}$

2.4 Markierung → GeoGebra

$A = \int_{-3}^{-1}(g_1(x)-f(x))\,dx = \int_{-3}^{-1}\left(\frac{2}{x}-2\cdot\frac{1+\ln(x^2)}{x}\right)dx$

$= \int_{-3}^{-1}\left(\frac{2-2-2\ln(x^2)}{x}\right)dx = \int_{-3}^{-1}\frac{-2\ln(x^2)}{x}\,dx = -2\int_{-3}^{-1}\frac{\ln(x^2)}{x}\,dx$

$= -2\left[\ln^2(-x)\right]_{-3}^{-1} = -2\left(\overset{=0}{\ln^2(1)} - \ln^2(3)\right) = 2\ln^2(3)$ $(\approx 2{,}41)$

2.5 $g_1(x) = \frac{2}{x}$; $g_1'(x) = -\frac{2}{x^2}$; Tangente an der Stelle $x_P = k$

$m_T = g_1'(k) = -\frac{2}{k^2}$; $y_P = g_1(k) = \frac{2}{k}$; $x_P = k$

$t = y_P - m x_P = \frac{2}{k} - \left(-\frac{2}{k^2}\right)\cdot k = \frac{2}{k}+\frac{2}{k} = \frac{4}{k}$; $t_k(x) = -\frac{2}{k^2}x + \frac{4}{k}$

Dreieck : NST $-\frac{2}{k^2}x + \frac{4}{k} = 0 \Leftrightarrow x_0 = 2k$ (≙ Breite) (≙ g)

$A_\Delta = \frac{1}{2}g\,h = \frac{1}{2}\cdot 2k\cdot\frac{4}{k} = 4$ unabh. von k